TO THE MEMORY OF K. SELIVANOV

An Exact Relation for $\mathcal{N} = 1$ Orientifold Field Theories with Arbitrary Superpotential

A. Armoni [a], A. Gorsky [b], M. Shifman [c]

[a] *Department of Physics, Theory Division*
CERN, CH-1211 Geneva 23, Switzerland

[b] *Institute of Theoretical and Experimental Physics, Moscow 177259, Russia*

[c] *William I. Fine Theoretical Physics Institute, University of Minnesota,*
Minneapolis, MN 55455, USA

Abstract

We discuss a nonperturbative relation for orientifold parent/daughter pairs of supersymmetric theories with an arbitrary tree-level superpotential. We show that super-Yang-Mills (SYM) theory with matter in the adjoint representation at $N \to \infty$, is equivalent to a SYM theory with matter in the antisymmetric representation and a related superpotential. The gauge symmetry breaking patterns match in these theories too. The moduli spaces in the limiting case of a vanishing superpotential are also discussed. Finally we argue that there is an exact mapping between the effective superpotentials of two finite-N theories belonging to an orientifold pair.

1

1 Introduction

Many people hope to extract lessons from supersymmetric (SUSY) theories that could teach us about nonperturbative behavior of theories with less SUSY, hopefully, without SUSY at all. A considerable progress was achieved in this direction. It was discovered that theories related by orbifold or orientifold projections are perturbatively planar equivalent [1, 2, 3, 4]. Extension of the perturbative equivalence to the nonperturbative level was elaborated in [5, 6]. While orbifold theories generally speaking do not enjoy nonperturbative planar equivalence [7, 8], orientifold theories do [6]. Planar equivalence means that two theories from a given "parent-daughter" pair have identical behavior at large N in common sectors. The definition of the "common sector" is given in the original paper [6] or in the review paper [9].

In this letter we focus on a specific example of an orientifold parent-daughter *supersymmetric* pair. Starting from the parent softly broken $\mathcal{N} = 2$ Yang-Mills theory with matter in the adjoint representation, and a generic superpotential, we compare it at $N \to \infty$ with a daughter super-Yang-Mills (SYM) theory with matter in the two-index antisymmetric representation. For quadratic superpotentials such a comparison was carried out in [9], with the conclusion that nonperturbative planar equivalence does take place in the common sector of the both theories. The common sector includes such non-holomorphic data as, say, the mass spectra. Here we will consider *arbitrary* superpotentials but compare *only chiral data*.

We will argue that, if classical superpotentials in both theories coincide,[1] then these theories are planar equivalent at the nonperturbative level in the chiral sector.

More concretely, we will show that the *effective* superpotentials in these theories coincide, implying that their holomorphic sectors are equivalent. Furthermore, we will demonstrate that the gauge symmetry breaking patterns coincide in these two theories too. The choice of the theories above is dictated by the ability to treat them exactly using approaches based on the matrix model [10] or the generalized Konishi anomalies in the holomorphic sector [11, 12].

First, we demonstrate that the effective superpotentials coincide in the

---

[1]The statement of coincidence is rather sloppy. The required relationship between the superpotentials in the orientifold pair is described in more accurate terms in Sect. 3.

two theories. To this end we use equivalence [13] of the matrix model expansion on the one hand, and calculation of field-theory loops in an effective background on the other hand. This is sufficient to prove the equality of the effective superpotentials in the planar limit.

To treat the symmetry breaking pattern we consider loop equations which can be derived either from the matrix model or from the generalized Konishi anomalies. Analysis of the loop equations provides us with the proper identifications of the field theory resolvents which amounts to establishing the symmetry breaking pattern. We exploit some results concerning anti-symmetric matter discussed previously in Refs. [14, 15, 16, 17, 18].

In addition, we will discuss the case of a vanishing tree-level superpotential, namely, correspondence between the large N moduli space of $\mathcal{N} = 2$ SYM theory and its orientifold daughter.

Finally and most importantly, we will consider a relation between the parent and the daughter theories at *finite N*. We will show that there exists an exact mapping between the effective superpotential of the two theories.

2 Planar equivalence and orientifold field theories

The idea of planar equivalence was introduced in Ref. [1]. It states that two *distinct* gauge theories coincide at large N in a certain sector. The original implementation of this idea was in the context of orbifolding. While at the perturbative level the planar equivalence does hold for orbifold field theories, it is not valid at the non-perturbative level [7, 8].

The status of the orientifold field theories is different. In this case both perturbative and nonperturbative proofs of equivalence exist; they were given in Ref. [6]. In this section we summarize main points of the proof. The reader can find a more detailed discussion in [6] and, especially, in the review paper [9].

The prime example of an orientifold pair is a U(N) gauge theory with matter in the adjoint representation and a U(N) gauge theory with matter in the two-index antisymmetric representation (in the present paper we consider the supersymmetric version of the former orientifold pair).

In the 't Hooft double index notation, the adjoint representation is pre-

sented by two lines with arrows pointing in the opposite directions, whereas for the antisymmetric representation the arrows on the two lines point in the same direction. In [6] it was shown that for planar graphs it is possible to flip the orientation of one of the arrows carrying color flow in the matter loops, without changing the value of any planar Feynman. This is a perturbative proof of the planar equivalence between the two theories.

In the same paper [6] a non-perturbative proof was given as well. The main idea is that the partition functions of the two distinct theories coincide at large N *before* integration over the gluon field. To this end it was demonstrated that the determinants in the two theories become identical at large N,

$$\lim_{N \to \infty} \frac{\det(i \not\partial + \not\!A^a T^a_{adj})}{\det(i \not\partial + \not\!A^a T^a_{anti})} = 1 \, . \tag{1}$$

This is sufficient to establish the nonperturbative equivalence between two theories.

In the present paper we focus on a supersymmetric pair, with a nonvanishing superpotential. We demonstrate the equivalence of the two SUSY theories in the chiral sector by showing that the *effective* superpotentials in the two theories coincide at large N.

3 Equivalence of the effective superpotentials

To begin with, let us define the Lagrangian of the parent $SU(N)$ theory with adjoint matter,[2]

$$\begin{aligned}
\mathcal{L}_P &= \frac{1}{2g^2} \int d^2\theta \, \mathrm{Tr}_F \, W^2 + \text{h.c.} \\
&+ \int d^2\theta d^2\bar\theta \, \mathrm{Tr}_{Ad} \bar\Phi e^V \Phi + \left(\int d^2\theta \, \mathcal{W}_P(\Phi) + \text{h.c.} \right) ,
\end{aligned} \tag{2}$$

where Φ is a chiral superfield in the adjoint and $\mathcal{W}_P$ is a superpotential which will be assumed even in Φ.

---

[2]The subscript F means that the trace is taken in the fundamental representation, $\mathrm{Tr}_F \, W^2 \equiv (1/2) W^a W^a$ while W is defined in such a way that $W_\alpha = i\lambda_\alpha + ...$

The Lagrangian of the daughter orientifold theory is

$$\mathcal{L}_D \;=\; \frac{1}{2g^2} \int d^2\theta \, \mathrm{Tr}_F \, W^2 + \text{h.c.}$$

$$+ \; \int d^2\theta \, d^2\bar\theta \, \mathrm{Tr}_{Anti} \left(\bar\chi e^V \chi + \bar\eta e^V \eta \right) + \left(\int d^2\theta \, \mathcal{W}_D(\chi\eta) + \text{h.c.} \right), \quad (3)$$

where χ, η are antisymmetric chiral superfields of the type $\chi^{[ij]}$, $\eta_{[ij]}$. We assume the tree-level superpotentials to have similar structure, namely,

$$\mathcal{W}_P(\Phi) = \sum_{k=1}^{k_*} g_k \, \mathrm{Tr}_{Ad} \, (\Phi^2)^k \,, \qquad \mathcal{W}_D(\chi\eta) = \sum_{k=1}^{k_*} g_k \, \mathrm{Tr}_{Anti} \, (\chi\eta)^k \,, \quad (4)$$

with the same coefficients g_k, $k = 1, 2 ..., k_*$ where k_* does not grow with N.

Turn now to the effective superpotentials in both theories. The simplest way to derive the effective superpotentials is to follow Ref. [13] where it was shown that the effective superpotentials can be calculated perturbatively from the following effective actions

$$\int dx d^2\theta \, \left[\bar\Phi (\Delta - iW_\alpha D_\alpha)\Phi + \mathcal{W}_P(\Phi) \right] \,, \quad (5)$$

$$\int dx d^2\theta \, [\bar\chi (\Delta - iW_\alpha D_\alpha)\chi + \bar\eta (\Delta - iW_\alpha D_\alpha)\eta + \mathcal{W}_D(\chi, \eta)] \,. \quad (6)$$

Here the chiral superfield W_α (the gauge field strength tensor) must be treated as a fixed constant background with

$$S = \frac{1}{32\pi^2} \mathrm{Tr}_F W^2 \,.$$

In the planar limit all graphs determining $\mathcal{W}_{\text{eff}}(S, g_k)$ are the same, with reversion of the color flow direction on one of two lines forming the loop. Correspondingly, the result of their calculation is the same in the parent and daughter theories at $N \to \infty$, much in the same way as in Ref. [6]. Actually, similar arguments with no reference to the orientifold pair were discussed previously in [14].

4 The equivalence of the symmetry breaking patterns

Having established the equivalence of the effective superpotentials we pass to the symmetry breaking pattern. To this end let us invoke another approach based on the generalized Konishi anomalies. It was shown that they play a crucial role in derivation of the Riemann surface which governs the chiral sector of the theory and is equivalent to the set of loop equations in the matrix model.

The generalized Konishi anomalies follow from variation of the chiral field with a function $f(\Phi, W)$,

$$\Phi \to e^f \Phi.$$

In this way one gets

$$\bar{D}^2 J_f = \mathrm{Tr}\left[f(\Phi, W)\frac{\partial \mathcal{W}(\Phi)}{\partial \Phi} \right] + \sum_{ijkl} A_{ijkl}\frac{\partial f(\Phi, W)_{ij}}{\partial \Phi_{kl}} \tag{7}$$

where

$$J_f = \mathrm{Tr}\,\bar{\Phi}e^{adV}f(\Phi, W), \tag{8}$$

and the tensor A_{ijkl} depends on the representation of the matter field as follows:

$$A_{ijkl} = \frac{1}{32\pi^2}[W_\alpha, [W^\alpha, T_{lk}]]_{ji}. \tag{9}$$

Here the generators of the gauge group T_{lk} are taken in the corresponding representations. It is convenient to introduce the resolvents

$$T(z) = \mathrm{Tr}_{Ad}\frac{1}{z-\Phi}, \qquad R(z) = \mathrm{Tr}_{Ad}\frac{W^2}{z-\Phi}. \tag{10}$$

Taking the function f in the generalized current first to be $T(z)$ and then $R(z)$ one derives the following chiral ring relations:

$$R^2(z) = \mathcal{W}'R(z) + \frac{1}{4}q(z),$$

$$2R(z)T(z) = \mathcal{W}'(z)T(z) + p(z), \tag{11}$$

where $q(z)$, $p(z)$ are polynomials of degree $(n-1)$ if $\mathcal{W}'(z)$ is a polynomial of degree n.

To get the symmetry breaking pattern one has to calculate the integrals over the resolvent $T(z)$ over the A_i cycles over the Riemann surface defined by the chiral ring,

$$\oint_{A_i} T(z) = N_i \,. \tag{12}$$

Once N_i are found one can say that the gauge group is broken as

$$\mathrm{SU}(N) \to \prod_i \mathrm{SU}(N_i) \,. \tag{13}$$

Hence, to compare the symmetry breaking patterns we have to compare the Riemann surfaces as well as the resolvents in the parent/daughter pair. The chiral ring relation for the $\mathrm{SU}(N)$ theory with the antisymmetric matter has to be found from the generalized Konishi anomaly or the matrix model. The calculation of the generalized anomaly [14] amounts to the spectral curve in the daughter theory, which coincides with the one for the parent theory in the planar limit. Note that is it convenient for our purpose to use slightly unconventional resolvents,

$$R_D(z) \;=\; \mathrm{Tr}_{Anti} \, \frac{zW^2}{z^2 - \chi\eta} \,, \tag{14}$$

$$T_D(z) \;=\; \mathrm{Tr}_{Anti} \, \frac{z}{z^2 - \chi\eta} \,. \tag{15}$$

Such resolvents respect the nonanomalous $\mathrm{U}(1)_V$ in the daughter theory, which results in the fact that only the products $\chi\eta$ could develop vacuum expectation values. The equations for these resolvents derived from the generalized Konishi anomalies are [34]

$$R_D^2(z) \;=\; \mathcal{W}' R_D(z) + \frac{1}{4} \, q(z) \,, \tag{16}$$

$$2R_D(z)T_D(z) \;=\; \mathcal{W}'(z)T_D(z) + \frac{2}{z} R_D(z) - 2R_D'(z) + p(z) \,. \tag{17}$$

Comparison with similar equations in the parent theory immediately indicates that the equations for $R(z)$ and $R_D(z)$ exactly coincide, while that for $T_D(z)$ has additional terms compared to the parent equation. However, these additional terms are subleading in N; hence the respective equations in the

two theories match at large N, which means that the symmetry breaking patterns coincide in the planar limit.

The example of the orientifold pair considered above is not unique. There are several other examples of pairs with equivalent perturbative behavior [4]. Geometrically they emerge from different orientations of the orientifold planes in the brane picture.

Let us comment on their nonperturbative equivalence. Consider an example of $SU(N)$ parent theory with adjoint matter and $Sp(2N)$ daughter theory with matter in the antisymmetric representation, or $SO(N)$ daughter with matter in the symmetric representation.

Let us exploit the exact duality found for $SU(N)$ theory with the adjoint matter and $Sp(2N)$ theory with the antisymmetric matter [15] (or, alternatively, $SO(N)$ theory with matter in the symmetric representation [20, 21]). The duality implies that the $SU(N)$ theory with the adjoint matter and superpotential $\mathcal{W}$ is nonperturbatively equivalent to $Sp(2N)$ theory provided the following relation between effective superpotentials takes place

$$\mathcal{W}^{\mathrm{Sp}(2N)} = \frac{1}{2}\,\mathcal{W}^{\mathrm{U}(N+2k_*)} + \mathrm{const}, \tag{18}$$

where k_* is the degree of the classical superpotentials. The breaking patterns also match in these two theories,

$$\mathrm{Sp}(2N) \rightarrow \prod_k \mathrm{Sp}(2N_k)\,,$$

$$\mathrm{U}(N + 2k_*) \rightarrow \prod_k \mathrm{U}(N_k + 2)\,. \tag{19}$$

At large N we can disregard small subleading factors if the degree of the classical superpotential k_* is restricted. The factor $1/2$ is related to the fact that Ref. [15] deals with only one matter field in the antisymmetric representation, without the conjugated one; hence, we have to double it to get total answer for the effective superpotential. Only in this case we have equal number of degrees of freedom in two theories and can discuss the orientifold pair. This concludes the argument that nonperturbative equivalence holds for this orientifold pair as well.

5 The moduli space in the case of a vanishing superpotential

The case of a vanishing tree-level superpotential is an interesting limiting case. The theory with adjoint matter becomes $\mathcal{N} = 2$ super-Yang-Mills theory. It admits a classical as well as a quantum moduli space. The exact metric on the moduli space was computed in the case of SU(2) by Seiberg and Witten [22]; later the analysis was generalized to arbitrary SU(N) in Refs. [23, 24].

If planar equivalence holds in the limit of a vanishing superpotential too, the orientifold daughter should admit the same large-N moduli space as the Seiberg-Witten $\mathcal{N} = 2$ theory. While this cannot be the case literally, a refined version of this statement is indeed valid.

The case of a vanishing superpotential is very subtle, as all fields are massless. Note that in the proof of non-perturbative equivalence a small mass m was needed as an infrared regulator [6]. It was assumed that the limit $m \to 0$ is smooth. In the present situation this limit need not be smooth, generally speaking. In the presence of a mass term there is no moduli space, whereas the theories develop a moduli space when $m = 0$. In order to demonstrate the subtlety of the limit $m \to 0$, we quote a result [9] for the ratio of matter condensates in the two theories at finite N,

$$\frac{m_D \langle \xi \eta \rangle}{m_P \langle \Phi^2 \rangle} = \frac{N-2}{N} (8\pi^2)^{\frac{4}{N^2} - \frac{4}{N}} \left(1 + \frac{1}{N}\right)^{\frac{4(N-1)}{N^2}} \left(\frac{\Lambda}{m}\right)^{2/N} . \tag{20}$$

It is clear that as the ratio m/Λ decreases, a critical value of N needed for the onset of the planar equivalence increases logarithmically,

$$N_* \sim \ln \frac{\Lambda}{m} . \tag{21}$$

Thus, our analysis in the following assumes the strict planar limit, $N = \infty$. The above example demonstrates that the limit $m \to 0$ and $1/N \to 0$ might not commute.

We turn now to the check of planar equivalence. The large-N limit in the case of a moduli space means that the matter vacuum expectation values should scale as $\sqrt{N}$, as we want to keep the W masses fixed (recall the the W mass is $M_W = gv$).

9

Let us analyze first the classical moduli space of the two theories. Let us assume, for simplicity that N is even (the analysis in the odd-N case is straightforward too). The classical moduli of $\mathcal{N} = 2$ are

$$u_k = \langle \operatorname{Tr}_{Ad} \Phi^k \rangle, \qquad k = 1, ..., N. \tag{22}$$

Since $\operatorname{Tr} \Phi = 0$ there are actually only $N - 1$ moduli. The classical moduli space of the daughter theory with the antisymmetric matter is

$$v_k = \langle \operatorname{Tr}_{Anti} (\chi\eta)^k \rangle, \qquad k = 1, ..., N/2. \tag{23}$$

Clearly the two moduli spaces do not match, even at the classical level.

Note, however, that the correspondence that we suggest is between the operators $(\chi\eta)^k$ and $(\Phi^2)^k$. In order to compare the two moduli spaces we should restrict ourselves to the subspace of $\mathcal{N} = 2$ SYM theory where the odd moduli are frozen, $\operatorname{Tr}_{Ad} \Phi^{2k+1} = 0$. Thus, we suggest a correspondence (or, actually, an equivalence) between the large-N classical moduli space of the orientifold daughter and the even moduli subspace of $\mathcal{N} = 2$ SYM theory.

Let us discuss now the quantum moduli space. The Seiberg-Witten curve of the $\mathcal{N} = 2$ theory is

$$y^2 + y(x^N + u_2 x^{N-2} + u_3 x^{N-3} + ... + u_N) + 1 = 0. \tag{24}$$

Another way of writing the Seiberg-Witten curve, that has a nice interpretation in terms of locations of D4 branes at $x = \{a_i\}$, is [25]

$$y^2 + y(x - a_1)(x - a_2)...(x - a_N) + 1 = 0. \tag{25}$$

The relation between the different parameterizations of the moduli, $\{a_i\}$ and $\{u_i\}$, is obtained by comparing the coefficients in front of x^i. In particular, $\sum a_i = 0$. Restricting ourselves to the case where the odd moduli are frozen,

$$u_{2k+1} = 0,$$

the Seiberg-Witten curve of this subspace can be written in terms of $N/2$ moduli $\{b_i\}$ as follows:

$$y^2 + y(x - b_1)(x + b_1)(x - b_2)(x + b_2)...(x - b_{N/2})(x + b_{N/2}) + 1 = 0, \tag{26}$$

or

$$y^2 + y(x^2 - b_1^2)(x^2 - b_2^2)...(x^2 - b_{N/2}^2) + 1 = 0. \tag{27}$$

10

This is just the expected Seiberg-Witten curve for the $SU(N)$ orientifold theory at large N.

It was obtained previously by brane techniques in Refs. [26, 27, 28]. The above result clearly reflects the presence of an orientifold plane in the brane picture: the branes are located on both sides of the orientifold at the following positions:

$$x = b_1, \ x = -b_1, \ x = b_2, \ x = -b_2, ..., \ x = b_{N/2}, \ x = -b_{N/2}.$$

This is also the solution for $SO(N)$ theories at large N.

Note that the equivalence of the Seiberg-Witten curves, at large-N, implies a coincidence of the BPS spectra of the two theories (up to $1/N$ corrections). A similar result for a SUSY/non-SUSY orientifold pair was already obtained in [6].

Let us emphasize that matching of the moduli space generically is true only for large N; there is an evident counterexample for $N = 3$. Indeed, in the parent $SU(3)$ $\mathcal{N} = 2$ theory there is a two-dimensional complex moduli space. At the same time, let us examine possible moduli space in the daughter $SU(3)$ $\mathcal{N} = 1$ theory with $N_f = 1$. First, note that there is no Higgs branch of the moduli space, which implies the $N_f > 1$ condition. On the other hand, it is known that there is no Coulomb branch of the moduli space in this theory either; hence, we have clear mismatch between the parent and daughter theories at $N = 3$.

Though we are interested only in the large N limit, we wish to comment on the odd N case. Here we have $(N - 1)/2$ moduli characterized by

$$v_k = \left\langle \operatorname{Tr}_{Anti}(\chi\eta)^k \right\rangle, \qquad k = 1, ..., (N - 1)/2. \tag{28}$$

It should be compared with the $\mathcal{N} = 2$ theory with odd N, where the odd moduli are frozen, $u_{2i+1} = 0, \ i = 1, ..., (N-1)/2$. The Seiberg-Witten curve in this case is

$$y^2 + yx(x^2 - b_1^2)(x^2 - b_2^2)...(x^2 - b_{(N-1)/2}^2) + 1 = 0. \tag{29}$$

Let us briefly compare our case with the *orbifold* (rather than orientifold) daughter. In the orbifold pair the $\mathcal{N} = 2$ parent theory gets mapped onto $\mathcal{N} = 1$ daughter with bifundamental matter. In the orbifold case comparison of the Seiberg-Witten curves of the pair was performed in [29] where it was

argued that in the large-N limit the curves match if all moduli in the parent theory which are not singlets under the orbifold group are set to zero. This is similar to the orientifold case under consideration. However, there is a difference between the two cases in the second ingredient of the solution to $\mathcal{N} = 2$ theory, namely the differential on the curve. In the orbifold case the differential gets changed resulting in a rescaling of the coupling constant in the daughter theory while in the orientifold case the coupling constants in the parent and daughter theories are the same.

In conclusion, it is possible to construct the large-N Seiberg–Witten curves of an orbifold/orientifold daughter theories by keeping the moduli invariant under the orbifold/orientifold action and projecting out the noninvariant moduli. An interesting question that we wish to pose here is: "can one obtain *all* field theories which admit a Seiberg-Witten curve and a brane realization in string theory by a suitable orbifold/orientifold projection?"

6   Relation for the orientifold pair at finite N

So far we discussed the equivalence in the orientifold pairs at large N. In this section we wish to make a much stronger statement of a relation between the two theories at finite N.

Let us consider first the N dependence in the U(N) theory with matter in the adjoint representation. In such a theory the effective superpotential takes the following form:

$$\mathcal{W}_{\text{eff}} = N(S \log S - S) + N\mathcal{W}_{\text{pert}}(S) , \tag{30}$$

where $\mathcal{W}_{\text{pert}}(S)$ (the "perturbative" part = the polynomial part of the effective action) follows from a matrix model integral. In particular, the whole action is proportional to N. The linear N dependence in the polynomial part of the effective action $\mathcal{W}_{\text{pert}}$ follows from the Dijkgraaf-Vafa prescription for calculating $\mathcal{W}_{\text{pert}}(S)$,

$$\mathcal{W}_{\text{pert}}(S) = \frac{\partial \mathcal{F}_0}{\partial S} , \tag{31}$$

where $\mathcal{F}_0 = \mathcal{F}_0(S, g_k)$.

Now let us ask what is the form of the resulting effective superpotential in the theory with the antisymmetric matter. As was argued above, at large N it must coincide with the effective superpotential (30) of the U(N) theory

coupled to the adjoint matter. Moreover, since for U(2) the antisymmetric representation is in fact a singlet, the matter decouples from the gauge part, and the effective superpotential must reduce to the Veneziano-Yankielowicz action [30]. The unique solution that meets those twin requirements is

$$\mathcal{W}_{\text{eff}} = N(S \log S - S) + (N - 2)\mathcal{W}_{\text{pert}}(S) \,. \tag{32}$$

Note a crucial point: nonpolynomial factors such as $N - 4/N$, that also vanish at $N = 2$, are ruled out, as they have no meaning in terms of the Ramond-Ramond fluxes in the brane picture.

The $N - 2$ factor in front of the effective superpotential in the theories that include orientifold planes, was already observed in [18, 19]. Thus, the knowledge of the effective superpotential in one of the theories automatically fixes the effective superpotential in the other. It is interesting to note that for SU(3), where the antisymmetric representation is equivalent to antifundamental, we can relate the actions of the theory with the adjoint and fundamental matter. This is quite remarkable!

As an example, let us consider a particular case with a tree-level *quartic* superpotential.[3] The effective superpotential for the theory with the adjoint matter was calculated in Ref. [31]; for the theory with the fundamental matter it was calculated in Ref. [32]. For SU(3) the antifundamental representation is equivalent to two-index antisymmetric. However, Refs. [31, 32] deal with U(N) group rather than SU(N), and, therefore, we need to translate their results from U(N) to SU(N). This translation, together with the relations

$$Q^i = \frac{1}{2}\epsilon^{ijk}\chi_{jk} \,, \qquad \bar{Q}_i = \frac{1}{2}\epsilon_{ijk}\eta^{[jk]} \,,$$

results in a relation between the tree-level couplings of the two theories. This result can be expressed as follows: the effective superpotentials in both theories take the same form, except for an anticipated overall factor (we use the notation of Ref. [31]),

$$\mathcal{W}_{\text{pert}}^{\text{adjoint}} = 3\mathcal{W}_{\text{pert}}^{\text{fundamental}} = -\sum_{k=1}^{\infty}\left(-\frac{3g}{2m^2}\right)^k S^{k+1}\frac{(2k-1)!}{k!(k+1)!} \,. \tag{33}$$

Let us note that the agreement between two SU(3) theories with quartic superpotentials on the one hand, and a mismatch in moduli in the limit

---

[3]We thank R. Argurio for bringing this example to our attention.

$\mathcal{W}_{\text{tree}} = 0$ on the other, presumably implies that in the process of switching off the superpotential in the daughter theory we arrive at a singularity point in the moduli space of the $\mathcal{N} = 2$ SU(3) parent theory.

Similar arguments can be applied to the case with general symmetry breaking patterns. Namely, if we consider the U$(N) \to \prod_i$ U(N_i) pattern we have the same twin requirements again. Indeed, we have already argued that at large N the symmetry breaking patterns in the orientifold pair match while in each $U(N_i)$ matter decouples if $N_i = 2$. Hence, we have a similar expression in the generic case,

$$\mathcal{W}_{\text{eff}} = \sum_i (N_i(S_i \log S_i - S_i) + (N_i - 2)\mathcal{W}_{\text{pert}}(S_i)) . \qquad (34)$$

Finally, it is rather clear that in the theory with the symmetric matter the effective superpotential is

$$\mathcal{W}_{\text{eff}} = N(S \log S - S) + (N + 2)\mathcal{W}_{\text{pert}}(S) . \qquad (35)$$

7 Discussion

In this paper we show that holomorphic data in the orientifold pair with $\mathcal{N} = \infty$ match in the planar limit. To an extent this follows from the possibility of deriving the effective potential perturbatively in the coupling constants using the formalism of the external constant composite background field S. Although these data do not cover the whole content of the theory, the nonperturbative planar equivalence we observe means that the domain wall tensions in two theories coincide. One could also consider the domain wall junctions saturating the central charge in the anticommutator of $\bar{Q}$ and Q. This central charge does not belong to the holomorphic sector but it involves the matter axial currents which can be mapped between the two theories. Hence, it is plausible that the tensions of the domain wall junctions coincide in these theories at large N as well.

Finally, we wish to comment on the finite-N relation that we established between the two SUSY theories. The relation amounts to a simple shift $N \to N - 2$. It could be extremely useful for phenomenology if this feature extends to certain quantities in the case of the SUSY/non-SUSY pair.

Note added. July 21, 2004.

After this work was completed and submitted for publication in Nuclear Physics, two related papers by Argurio and Landsteiner were posted on the electronic archive [33, 34]. These two works treat effective superpotentials in theories with antisymmetric matter, and both works support our assertion of planar equivalence.

References

[1] S. Kachru and E. Silverstein, Phys. Rev. Lett. **80**, 4855 (1998) [hep-th/9802183].

[2] A. E. Lawrence, N. Nekrasov and C. Vafa, Nucl. Phys. B **533**, 199 (1998) [hep-th/9803015].

[3] M. Bershadsky, Z. Kakushadze and C. Vafa, Nucl. Phys. B **523**, 59 (1998) [hep-th/9803076]; M. Bershadsky and A. Johansen, Nucl. Phys. B **536**, 141 (1998) [hep-th/9803249].

[4] Z. Kakushadze, Nucl. Phys. B **529**, 157 (1998) [hep-th/9803214]; Phys. Rev. D **58**, 106003 (1998) [hep-th/9804184]; M. Schmaltz, Phys. Rev. D **59**, 105018 (1999) [hep-th/9805218].

[5] M. J. Strassler, *On methods for extracting exact non-perturbative results in non-supersymmetric gauge theories,* hep-th/0104032.

[6] A. Armoni, M. Shifman and G. Veneziano, Nucl. Phys. B **667**, 170 (2003) [hep-th/0302163].

[7] A. Gorsky and M. Shifman, Phys. Rev. D **67**, 022003 (2003) [hep-th/0208073].

[8] D. Tong, JHEP **0303**, 022 (2003) [hep-th/0212235].

[9] A. Armoni, M. Shifman and G. Veneziano, *From super-Yang-Mills theory to QCD: Planar equivalence and its implications*, to be published in *From Fields to Strings: Circumnavigating Theoretical Physics*, The Ian Kogan Memorial Collection, Eds. M. Shifman, A. Vainshtein and J. Wheater (World Scientific, 2004), hep-th/0403071.

[10] R. Dijkgraaf and C. Vafa, Nucl. Phys. B **644**, 3 (2002) [hep-th/0206255].

[11] A. Gorsky, Phys. Lett. B **554**, 185 (2003) [hep-th/0210281].

[12] F. Cachazo, M. R. Douglas, N. Seiberg and E. Witten, JHEP **0212**, 071 (2002) [hep-th/0211170].

[13] R. Dijkgraaf, M. T. Grisaru, C. S. Lam, C. Vafa and D. Zanon, Phys. Lett. B **573**, 138 (2003) [hep-th/0211017].

[14] S. G. Naculich, H. J. Schnitzer and N. Wyllard, JHEP **0308**, 021 (2003) [hep-th/0303268].

[15] F. Cachazo, *Notes on supersymmetric* Sp(N) *theories with an antisymmetric tensor,* hep-th/0307063.

[16] P. Kraus, A. V. Ryzhov and M. Shigemori, JHEP **0305**, 059 (2003) [hep-th/0304138].

[17] L. F. Alday and M. Cirafici, JHEP **0305**, 041 (2003) [hep-th/0304119].

[18] K. Landsteiner, C. I. Lazaroiu and R. Tatar, JHEP **0311**, 044 (2003) [hep-th/0306236].

[19] K. Landsteiner, C. I. Lazaroiu and R. Tatar, JHEP **0402**, 044 (2004) [hep-th/0307182].

[20] K. Landsteiner and C. I. Lazaroiu, Phys. Lett. B **588**, 210 (2004) [hep-th/0310111].

[21] K. Intriligator, P. Kraus, A. V. Ryzhov, M. Shigemori and C. Vafa, *On low rank classical groups in string theory, gauge theory and matrix models,* hep-th/0311181.

[22] N. Seiberg and E. Witten, Nucl. Phys. B **426**, 19 (1994) [Erratum-ibid. B **430**, 485 (1994)] [hep-th/9407087].

[23] A. Klemm, W. Lerche, S. Yankielowicz and S. Theisen, Phys. Lett. B **344**, 169 (1995) [hep-th/9411048].

[24] P. C. Argyres and A. E. Faraggi, Phys. Rev. Lett. **74**, 3931 (1995) [hep-th/9411057].

[25] E. Witten, Nucl. Phys. B **500**, 3 (1997) [hep-th/9703166].

[26] C. Csaki, M. Schmaltz, W. Skiba and J. Terning, Phys. Rev. D **57**, 7546 (1998) [hep-th/9801207].

[27] J. Park, Nucl. Phys. B **550**, 238 (1999) [hep-th/9805029].

[28] K. Landsteiner, E. Lopez and D. A. Lowe, JHEP **9807**, 011 (1998) [hep-th/9805158].

[29] J. Erlich and A. Naqvi, JHEP **0212**, 047 (2002) [hep-th/9808026].

[30] G. Veneziano and S. Yankielowicz, Phys. Lett. B **113**, 231 (1982).

[31] H. Fuji and Y. Ookouchi, JHEP **0212**, 067 (2002) [hep-th/0210148].

[32] R. Argurio, V. L. Campos, G. Ferretti and R. Heise, Phys. Rev. D **67**, 065005 (2003) [hep-th/0210291].

[33] K. Landsteiner, *Konishi anomalies and curves without adjoints,* hep-th/0406220.

[34] R. Argurio, *Effective superpotential for U(N) with antisymmetric matter,* hep-th/0406253.